## CS 421 --- CPS Activity

| Manager | Keeps team on track |  |
| :--- | :--- | :--- |
| Recorder | Records decisions |  |
| Reporter | Reports to class |  |
| Reflector | Assesses team performance |  |

## Observations

Consider the following three programs:

```
1 fact 0 k = k 1
    1 sumList [] k = k 0
2 fact n k = fact ( }n-1)(\v >> k (n * v)) ( ) sumList (x:xs) = sumList xs (\v -> k (x + v))
1 prodList [] ks ka = ks 1
2 prodList (0:xs) ks ka = ka 0
3 prodList (x:xs) ks ka = prodList xs (\v -> ks (x * v)) ka
```

Problem 1) Which of the above calls to $k$, ka, or ks are in tail position?

Problem 2) Which of the above recursive calls are in tail position?

Problem 3) What is the role of $v$ in the above codes?

Problem 4) What is the role of ka and ks in the prodList code?

## Now It's Ruined

Consider the following three programs. They are not in CPS.

```
1 decList [] k = []
2 decList (x:xs) k = (x-1 : decList xs k)
1 maxk a b k = if a > b then k a else k b
2maxList [x] k = k x
3 maxList (x:xs) k = maxk x (maxList xs k) k
1 mink a b k = if a < b then k a else k b
2minList [x] k = k x
3minList (x:xs) k = maxList xs (\v -> maxk v x id)
```

Problem 5) For each of the programs above, describe why each fails to be in continuation passing style.

## Conversion

Here are the conversion rules.

$$
C \llbracket f \arg =e \rrbracket \Rightarrow f \arg k=C \llbracket e \rrbracket_{k}
$$

$$
\begin{aligned}
& C \llbracket a \rrbracket_{k} \Rightarrow k a \\
& C \llbracket f \arg \rrbracket_{k} \Rightarrow f \arg k \\
& \left.C \llbracket \operatorname{farg} \rrbracket_{k} \Rightarrow C \llbracket \arg \rrbracket_{(\lambda v . f v}\right) \text {, where } v \text { is fresh. } \\
& C \llbracket e_{1}+e_{2} \rrbracket_{k} \Rightarrow k\left(e_{1}+e_{2}\right) \\
& C \llbracket e_{1}+e_{2} \rrbracket_{k} \Rightarrow C \llbracket e_{1} \rrbracket_{(\lambda v->k(v+e 2))} \text { where } v \text { is fresh. } \\
& C \llbracket e_{1}+e_{2} \rrbracket_{k} \Rightarrow C \llbracket e_{1} \rrbracket_{\left(\lambda v_{1}->C \llbracket e_{2} \rrbracket_{\lambda v_{2}->k\left(v_{1}+v_{2}\right)}\right)} \text { where } v_{1} \text { and } v_{2} \text { are fresh. }
\end{aligned}
$$

Problem 6) The phrase " 'where $v$ is fresh" appears a lot here. Why do we need to be concerned about that?

Problem 7) A lot of these rules seem to have more than one form. How do you know which version to use?

## Convert To

Problem 8) Convert map to CPS. Assume $f$ is written in direct style.

1 map $f[]=[]$
2 map $f(x: x s)=f x: m a p h s$

Problem 9) Do it again, but this time assume $f$ is written in CPS and takes one continuation.

1 map f [] = []
2 map f (x:xs) = f x : map f xs

Problem 10) Convert the following code to CPS. Note: you will need a nested continuation to make this work.

1 min $\mathrm{a} b=$ if $\mathrm{a}<\mathrm{b}$ then a else b
$2 \min 4 a b c d=\min (m i n a b)(m i n c d)$

## Reordering Computations

On the off chance we have extra time, here's something to try.
Suppose you have a calculator which has an accumulator and a list of instructions. Add i adds ito the accumulator, and Sub i subtracts i from the accumulator.

```
1 data Calc = Add Integer
    | Sub Integer
        deriving (Eq,Show)
```

The only problem is that our accumulator cannot ever be negative! Use continuations to fix this. Here's the original calculator:

1 calc $x x=$ aux $0 x x$
where aux a [] = a
aux $a(($ Add $i): x s)=$ aux ( $a+i$ ) xs
aux a ((Sub i):xs) = aux (a-i) xs
Hint: you will need two continuations to make this work.

## CPS Activity--- Reflector's Report

| Manager | Keeps team on track |  |
| :--- | :--- | :--- |
| Recorder | Records decisions |  |
| Reporter | Reports to Class |  |
| Reflector | Assesses team performance |  |

1. What was a strength of your team's performance for this activity?
2. What could you do next time to increase your team's performance?
3. What insights did you have about the activity or your team's interaction today?
