Introduction to Higher Order Functions

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Objectives

- Explain the concept of *first class citizen*.
- Use sectioning and lambda to define anonymous functions.
- Change the behavior and interface of a function by using another function.

First Class Functions

An entity is said to be *first class* when it can be:

- **Assigned** to a variable, **passed** as a parameter, or **returned** as a result

Examples:

- APL: scalars, vectors, arrays
- C: scalars, pointers, structures
- C++: like C, but with objects
- HASKELL, LISP, OCAML: scalars, lists, tuples, functions

The Kind of Data a Program Manipulates Changes the Expressive Ability of a Program.

Defining Functions the Usual Way

Some HASKELL Functions

```
1 sqr a = a * a
2 hypotsq a b = sqr a + sqr b
```

Sample Run

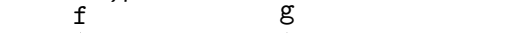
```
1 sqr :: Integer -> Integer
2 sqr :: Num a => a -> a
3 hypotsq :: Num a => a -> a -> a
4 Prelude> sqr 10
5 100
6 Prelude> hypotsq 3 4
7 25
```

Example: Compose

Example

```
inc x = x + 1
double x = x * 2
compose f g x = f (g x)
```

- Notice the function types.




```
compose :: (t1 -> t2) -> (t -> t1) -> t -> t2
Prelude> :t double
double :: Integer -> Integer
Prelude> double 10
20
Prelude> compose inc double 10
21
```

Example: Twice

- One handy function allows us to do something twice.
- You will see this function again!

Twice

```
1 twice f x = f (f x)
```

Here is a sample run ...

```
Prelude> :t twice
twice :: (t -> t) -> t -> t
Prelude> twice inc 5
7
Prelude> twice twice inc 4
```

Creating Functions: Lambda Form

- Functions do not have to have names.

```
\x -> x + 1
```

- The parts:
  - Backslash (a.k.a. *lambda*)
  - Parameter list
  - Arrow
  - Body of function

```
prelude> (\x -> x + 1) 41
42
```

Creating Functions: Partial Application

Standard Form vs. Anonymous Form

```
1 inc :: (Num t) => t -> t
2 inc a = a + 1
3 inc = \a -> a + 1
4
5 plus :: (Num t) => t -> t -> t
6 plus a b = a + b
7 plus = \a -> \b -> a + b
```

- What do you think we would get if we called `plus 1` ?

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6 plus a b = a + b
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```

- What do you think we would get if we called `plus 1` ?

```
1 inc = plus 1
```

η-equivalence

An Equivalence

$$\texttt{f} \equiv \texttt{\textbackslash x -> f x}$$

- Proof, assuming `f` is a function…

$$\texttt{f z} \equiv \texttt{(\textbackslash x -> f x) z}$$

These are Equivalent

```
1 plus a b = (+) a b
2 plus a = (+) a
3 plus = (+)
```

So are These

```
1 inc x = x + 1
2 inc = (+) 1
3 inc = (+1)
```

Curry and Uncurry

- Suppose you have a function `tplus` that takes a pair of integers and adds them.

```
tplus :: (Integer,Integer) -> Integer
tplus (a,b) = a + b
```

- But you really wish it took its arguments one at a time.
- There's a function `curry :: (a,b) -> c -> a -> b -> c` that will convert it for you! See if you can write it.