Lambda Calculus

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Objectives

You should be able to ...

The purposes of this lecture is to introduce lambda calculus and explain the role it has in programming languages.

- Explain the three constructs of λ-calculus.
- Given a syntax tree diagram, write down the equivalent λ-calculus term.
- Perform a beta-reduction.

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abc \\
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- Used extensively in research. The "little white mouse" of computer science.
- We can implement this trivially in Haskell.
  $\lambda x.x$ = `\x -> x`.

Examples

$\lambda x.x$ “The identity”

$\lambda x.xx$ “Delta”

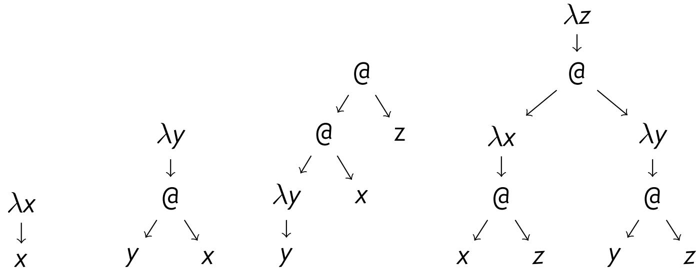
$\lambda ab.fabxy$

$(\lambda ab.fab)xy$

$(\lambda a.\lambda b.fab)xy$

$(\lambda fx.xf)(\lambda g.gx)(\lambda f.f)zy$

Syntax Trees

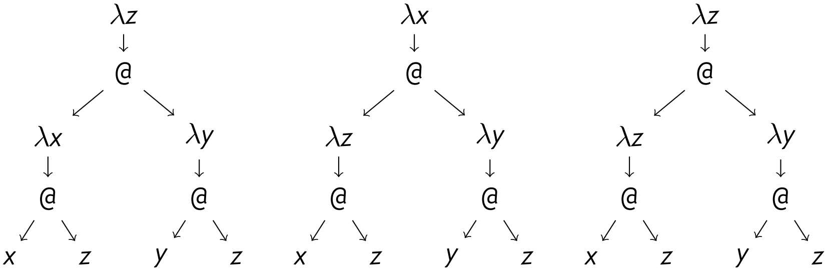


Example 1	Example 2	Example 3	Example 4
$\lambda x.x$	$\lambda y.yx$	$(\lambda y.y)xz$	$\lambda z.(\lambda x.xz)(\lambda y.yz)$

Bound and Free

- The λ creates a *binding*.
- An occurance of the the variable inside the function body is said to be *bound*.
- Bound variables occur "under the λ" that binds them.

Examples: Where are the free variables? To which lambdas are bound variables bound?



$\lambda z.(\lambda x.xz)(\lambda y.yz)$ $\quad$ $\lambda x.(\lambda z.xz)(\lambda y.yz)$ $\quad$ $\lambda z.(\lambda z.xz)(\lambda y.yz)$

Function Application

$$(\lambda x.M)N \quad \mapsto \quad [N/x]M$$

$$[N/x]\, y = y$$
$$[N/x]\, x = N$$
$$[N/x]\, (M\ P) = ([N/x]M\ [N/x]P)$$
$$[N/x]\, (\lambda y.M) = \lambda y.[N/x]M$$
$$[N/x]\, (\lambda x.M) = \lambda x.M$$