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Objectives

Monads

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- Know the three monad laws.
- ► Know the syntax for declaring monadic operations.
- ▶ Be able to give examples using the Maybe and List monads.





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Introducing Monads

- Monads are a way of defining computation.
- ► A *monad* is a container type *m* along with two functions:
 - ▶ return :: a -> m a
 - ▶ bind :: m a -> (a -> m b) -> m b
 - ► In HASKELL, bind is written as >>=
- ► These functions must obey three laws:

Left identity return a >>= f is the same as f a.

Right identity m >>= return is the same as m.

Associativity (m >>= f) >>= g is the same as m >>= ($x \rightarrow f x >>= g$).

Understanding Return

- ▶ return :: a -> m a
- ▶ The return keyword takes an element and puts it into a monad.
- ► This is a one-way trip!
- ▶ Very much like pure in the applicative type class.

```
instance Monad Maybe where
return a = Just a
instance Monad [] where
return a = [a]
instance Monad (Either a) where
return a = Right a
```





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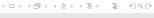
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Understanding Bind

- ► All the magic happens in bind.
- ▶ bind :: m a -> (a -> m b) -> m b
 - ► The first argument is a monad.
 - ► The second argument takes a monad, unpacks it, and repackages it with the help of the function argument.
 - Exactly how it does that is the magic part.

Bind for Maybe

```
1 Nothing >>= f = Nothing
2 (Just a) >>= f = f a
3   -- Remember that f returns a monad
```





- ▶ They are similar to continuations, but even more powerful.
- ► They are also related to the applicative functors from last time.
- Consider this program:

```
inc1 a = a + 1
2r1 = inc1 <$> Just 10 -- result: Just 11
3r2 = inc1 <$> Nothing -- result: Nothing
```

But what if we have functions like this?

```
inc2 a = Just (a+1)
zrecip a | a =/ 0 = Just (1/a)
def otherwise = Nothing
```

How can we pass a Nothing to it? How can we use what we get from it?



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Notice the Pattern

- ► Applicatives take the values out of the parameters, run them through a function, and then repackage the result for us.
- ▶ The functions have no control: the applicative makes all the decisions.
- ▶ Monads let the functions themselves decide what should happen.

A Calculator, with Monads

- ► Okay, the above code works, but here's a better way.
- ▶ First define functions lift to convert a function to monadic form for us!

These are part of Control. Monad:





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Lifting

- ▶ fail is another useful monadic function, defined in the MonadFail typeclass.
- ► Here it's defined as Nothing.

The Maybe Monad

► Here is the complete monad definition for Maybe.

Maybe Monad

```
instance Monad Maybe where
return = Just

(>>=) Nothing f = Nothing
(>>=) (Just a) f = f a

fail s = Nothing
```



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Example with Maybe

```
Prelude> minc (Just 10)
Just 11
Prelude> madd (minc (Just 10)) (Just 20)
Just 31
Prelude> mdiv (minc (Just 10)) (minc (Just 2))
Just 3
Prelude> minc (mdiv (minc (Just 10)) (minc (Just 2)))
Just 4
Prelude> minc (mdiv (minc (Just 10)) (Just 0))
Nothing
```

The List Monad

Lists can be monads too. The trick is deciding what bind should do.

List Monad

```
instance Monad [] where
return a = [a]

(>>=) [] f = []
(>>=) xs f = concatMap f xs

fail s = []
```

▶ Note that we do not have to change *anything* in our lifted calculator example!





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Example with List

```
Prelude> minc [1,2,3]
[2,3,4]
Prelude> madd [1,2,3] [10,200]
[11,201,12,202,13,203]
Prelude> minc (mdiv [10] [0])
[]
Prelude> minc (mdiv [10] [0,2,5])
[5,2]
```

