Fixing Non-LL Grammars

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Objectives

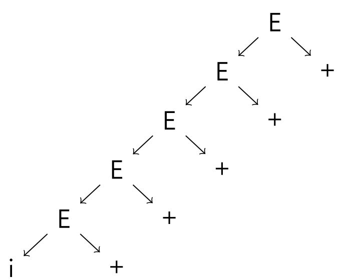
Last time we talked about grammars cannot be parsed using LL. Here we will try to fix them.

- Eliminate left recursion and mutual left recursion from a grammar.
- Eliminate common prefixes from a grammar.
- Detect and eliminate conflicts with FIRST and FOLLOW sets.

The Idea

Consider deriving `i++++` from the following grammar:

$E \rightarrow E +$	"We can have as many +s as we want *at the end* of the sentence."
$E \rightarrow i$	"The first word must be an `i`."



More Complicated Example

Consider the following grammar. What does it mean?

$$B \rightarrow Bxy \mid Bz \mid q \mid r$$

- At the end can come any combination of `x y` or `z`.
- At the beginning can come `q` or `r`.

Eliminating the Left Recursion

We can rewrite these grammars $E \rightarrow E + \mid i$, $B \rightarrow Bxy \mid Bz \mid q \mid r$

using the following transformation:

- Productions of the form $S \rightarrow \beta$ become $S \rightarrow \beta S'$.
- Productions of the form $S \rightarrow S\alpha$ become $S' \rightarrow \alpha S'$.
- Add $S' \rightarrow \epsilon$.

Result:

$$E \rightarrow iE'$$
$$E' \rightarrow +E' \mid \epsilon$$
$$B \rightarrow qB' \mid rB'$$
$$B' \rightarrow xyB' \mid zB' \mid \epsilon$$

Mutual Recursions!

Things are slightly more complicated if we have mutual recursions.

$$A \rightarrow Aa \mid Bb \mid Cc \mid q$$
$$B \rightarrow Ax \mid By \mid Cz \mid rA$$
$$C \rightarrow Ai \mid Bj \mid Ck \mid sB$$

How to do it:

- Take the first symbol (A) and eliminate immediate left recursion.
- Take the second symbol (B) and substitute left recursions to A. Then eliminate immediate left recursion in B.
- Take the third symbol (C) and substitute left recursions to A and B. Then eliminate immediate left recursion in C.

Left Recursion Example

Here is a more complex left recursion.

$A \rightarrow Aa \mid Bb \mid Cc \mid q$

$B \rightarrow Ax \mid By \mid Cz \mid rA$

$C \rightarrow Ai \mid Bj \mid Ck \mid sB$

First we eliminate the left recursion from A.

$A \rightarrow Aa \mid Bb \mid Cc \mid q$

This is the result:

$A \rightarrow BbA' \mid CcA' \mid qA'$

$A' \rightarrow aA' \mid \epsilon$

Left Recursion Example, 2

We substituting in the new definition of A, and now we will work on the B productions.

$A \rightarrow BbA' \mid CcA' \mid qA'$

$A' \rightarrow aA' \mid \epsilon$

$B \rightarrow Ax \mid By \mid Cz \mid rA$

$C \rightarrow Ai \mid Bj \mid Ck \mid sB$

First, we eliminate the "backward" recursion from B to A.

Start: $B \rightarrow Ax$

Result: $B \rightarrow BbA'x \mid CcA'x \mid qA'x$

Left Recursion Example, 3

$A \rightarrow BbA' \mid CcA' \mid qA'$
$A' \rightarrow aA' \mid \epsilon$
$B \rightarrow BbA'x \mid CcA'x \mid qA'x \mid By \mid Cz \mid rA$
$C \rightarrow Ai \mid Bj \mid Ck \mid sB$

Now we can eliminate the simple left recursion in B:

$B \rightarrow CcA'xB' \mid qA'xB' \mid CzB' \mid rAB'$
$B' \rightarrow bA'xB' \mid yB' \mid \epsilon$

Left Recursion Example, 4

$A \rightarrow BbA' \mid CcA' \mid qA'$
$A' \rightarrow aA' \mid \epsilon$
$B \rightarrow CcA'xB' \mid qA'xB' \mid CzB' \mid rAB'$
$B' \rightarrow bA'xB' \mid yB' \mid \epsilon$
$C \rightarrow Ai \mid Bj \mid Ck \mid sB$

Now production C: First, replace left recursive calls to A ...

$$C \rightarrow B\, bA'i \mid CcA'i \mid qA'i \mid B\, j \mid Ck \mid sB$$

Next, replace left recursive calls to B (this gets messy) ...

$$\begin{aligned} C \rightarrow\ & CcA'xB'\, bA'i \mid qA'xB'\, bA'i \mid CzB'\, bA'i \mid rAB'\, bA'i \mid \\ & CcA'xB'\, j \mid qA'xB'\, j \mid CzB'\, j \mid rAB'\, j \\ & CcA'i \mid qA'i \mid Ck \mid sB \end{aligned}$$

Left Recursion Example, 5

Reorganizing C, we have:

$$\begin{aligned} C \rightarrow\ & qA'xB'bA'i \mid rAB'bA'i \mid qA'xB'j \mid rAB'j \mid qA'i \mid sB \\ & CcA'xB'bA'i \mid CzB'bA'i \mid CcA'xB'j \mid CzB'j \mid CcA'i \mid Ck \end{aligned}$$

Eliminating left recursion gives us:

$$\begin{aligned} C \rightarrow\ & qA'xB'bA'iC' \mid rAB'bA'iC' \mid qA'xB'jC' \\ & \mid rAB'jC' \mid qA'iC' \mid sBC' \\ C' \rightarrow\ & cA'xB'bA'iC' \mid zB'bA'iC' \mid cA'xB'jC' \\ & \mid zB'jC' \mid cA'iC' \mid kC' \mid \epsilon \end{aligned}$$

The Result

Our final grammar:

$$\begin{aligned} A &\rightarrow BbA' \mid CcA' \mid qA' \\ A' &\rightarrow aA' \mid \epsilon \\ B &\rightarrow CcA'xB' \mid qA'xB' \mid CzB' \mid rAB' \\ B' &\rightarrow bA'xB' \mid yB' \mid \epsilon \\ C &\rightarrow qA'xB'bA'iC' \mid rAB'bA'iC' \mid qA'xB'jC' \\ &\quad \mid rAB'jC' \mid qA'iC' \mid sBC' \\ C' &\rightarrow cA'xB'bA'iC' \mid zB'bA'iC' \mid cA'xB'jC' \\ &\quad \mid zB'jC' \mid cA'iC' \mid kC' \mid \epsilon \end{aligned}$$

Beautiful, isn't it? I wonder why we don't do this more often?

- Disclaimer: If there is a cycle ($A \rightarrow^{+} A$) or an epsilon production ($A \rightarrow \epsilon$) then this technique is not guaranteed to work.

Common Prefix

This grammar has common prefixes.

$$A \to xyB \mid CyC \mid q$$
$$B \to zC \mid zx \mid w$$
$$C \to y \mid x$$

To check for common prefixes, take a nonterminal and compare the FIRST sets of each production.

Production	FirstSet
$A \to xyB$	$\{x\}$
$A \to CyC$	$\{x, y\}$
$A \to q$	$\{q\}$

If we are viewing an A, we will want to look at the next token to see which A production to use. If that token is x, then which production do we use?

Left Factoring

If $A \to \alpha\beta_1 \mid \alpha\beta_2 \mid \gamma$ we can rewrite it as $A \to \alpha A' \mid \gamma$, $A' \to \beta_1 \mid \beta_2$.

So, in our example:

$A \to xyB \mid CyC \mid q$
$B \to zC \mid zx \mid w$
$C \to y \mid x$

becomes

$A \to xA' \mid q \mid yyC$
$A' \to yB \mid yC$
$B \to zB' \mid w$
$B' \to C \mid x$
$C \to y \mid x$.

Sometimes you'll need to do this more than once. Note that this process can destroy the meaning of the nonterminals.

Epsilon Productions

- Epsilon productions have to be handled with care.

$$A \to Bc \mid x$$
$$B \to c \mid \epsilon$$

Is this LL?

Epsilon Productions

$$A \to Bc \mid x$$
$$B \to c \mid \epsilon$$

- $FOLLOW(B) = \{c\}$, and $FIRST(B) = \{c\}$, so we have a conflict when trying to parse B.
- We can substitute the B rule into the A rule to fix this ...
- Be sure to check if you have introduced a common prefix though!

$$A \to cc \mid c \mid x \quad \Rightarrow \quad A \to cA' \mid x, \quad A' \to c \mid \epsilon$$