LR Parsing

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Objectives

You should be able to ...

- Explain the difference between an LL and LR parser.
- Generate the finite state machine from an LR grammar.
- Use the state machine to detect ambiguity.

Further reading: See Dragon Book §4.x.

What Is LR Parsing?

- What is an LR parser?
  - An LR parser uses a **L**eft-to-right scan and produces a **R**ightmost derivation.
  - A.k.a. bottom-up parsing
  - Uses a *push-down automata* to do the work.
- There are four actions.

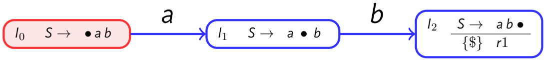
Shift	Consume a token from the input.
Reduce	Build a tree from components.
Goto	Jump to a different state, after a reduce.
Accept	Signal that we're done.

Shifting

Shifting involves three steps.

1. Consume a token from the input.
2. Push the token and the current state to the stack.
3. Go to the next state.

Example:



Grammar $S \rightarrow a\, b$

Input • a b $

Stack (empty)

Current State 0

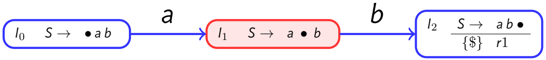
We will shift the a and then we go to state 1.

Shifting

Shifting involves three steps.

1. Consume a token from the input.
2. Push the token and the current state to the stack.
3. Go to the next state.

Example:



Grammar $S \to a\, b$

Input a ● b $

Stack 0, a

Current State 1

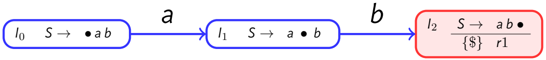
We will shift the b and then we go to state 2.

Shifting

Shifting involves three steps.

1. Consume a token from the input.
2. Push the token and the current state to the stack.
3. Go to the next state.

Example:



Grammar	$S \to a\ b$
Input	a b ● $
Stack	0, a, 1, b
Current State	2

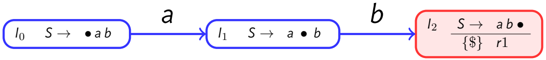
What should happen now?

Reducing

Reducing involves three steps.

1. Pop the tokens and states from the stack. (How many?)
2. Return to the last state popped.
3. Construct a new tree from the popped tokens.

Example:



Grammar	$S \to a\, b$
Input	a b ● $
Stack	0, a, 1, b
Current State	2

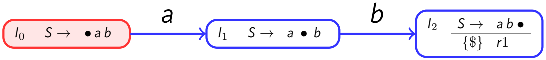
We are ready to reduce.

Reducing

Reducing involves three steps.

1. Pop the tokens and states from the stack. (How many?)
2. Return to the last state popped.
3. Construct a new tree from the popped tokens.

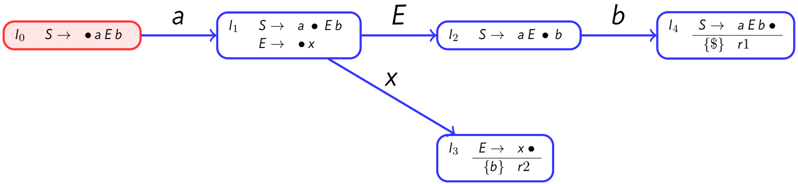
Example:



Grammar	$S \to a\, b$
Input	a b ● $
Stack	
Current State	0

Now we have an S tree. Go To or Accept could happen here.

A More Complex Example



Grammar

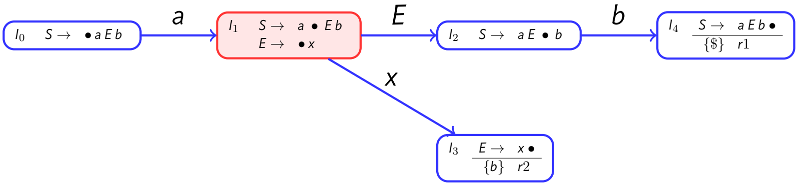
$$S \rightarrow a\, E\, b$$
$$E \rightarrow x$$

Input ● a x b $

Stack (Empty)

Current State 0

A More Complex Example



Grammar

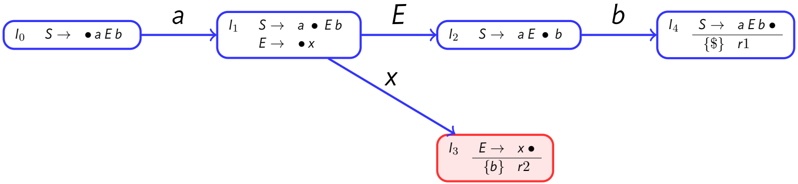
$$S \rightarrow a\, E\, b$$
$$E \rightarrow x$$

Input a ● x b $

Stack 0,a

Current State 1

A More Complex Example



Grammar

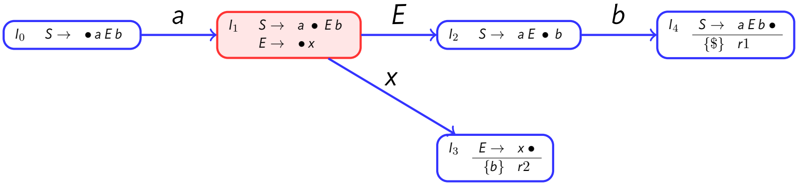
$$S \to a\,E\,b$$
$$E \to x$$

Input a x ● b $

Current State 4

Stack 0,a,1,x

A More Complex Example



Grammar

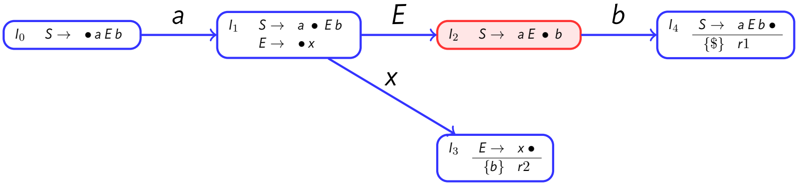
$$S \rightarrow a E b$$
$$E \rightarrow x$$

Stack 0,a

Input a x ● b $

Current State 1

A More Complex Example



Grammar

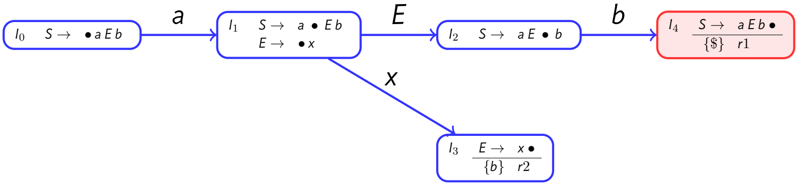
$$S \to a\,E\,b$$
$$E \to x$$

Input a x ● b $

Stack 0,a, 1, E / x

Current State 2

A More Complex Example



Grammar

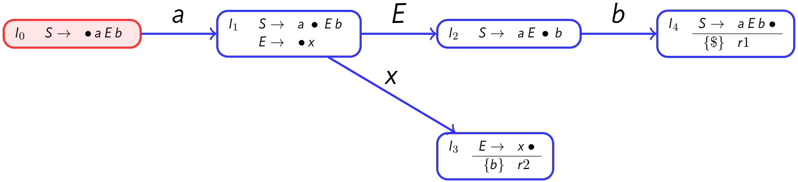
$$S \to a\, E\, b$$
$$E \to x$$

Input a x b ● $

Current State 3

E
|
Stack 0,a, 1, x , 2, b

A More Complex Example



Stack (Empty)

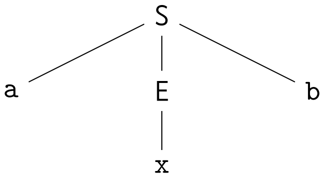
Grammar

$$S \rightarrow a\, E\, b$$
$$E \rightarrow x$$

Input a x b ● $

Current State 0

Now we have the result:



Representing the Automata

We will represent the automata using two tables.

Action Table Shift, Reduce n, Accept

Goto Table Destination State

The rows are the state numbers, the columns are the symbols.

The Algorithm

- To create the start state, add the *transitive closure* of the start symbol.

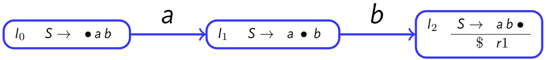
Example 1	Start	Example 2	Start
$S \rightarrow x\,S\,e$	$S \rightarrow \bullet\, x\,S\,e$	$S \rightarrow x\,S\,e$	
$\mid\ E\,x$	$\mid\ \bullet\, E\,x$	$\mid\ F\,x$	$S \rightarrow \bullet\, x\,S\,e$
$E \rightarrow a\,E$	$E \rightarrow \bullet\, a\,E$	$E \rightarrow a\,E$	$\mid\ \bullet\, F\,x$
$\mid\ F\,x$	$\mid\ \bullet\, F\,x$	$\mid\ F\,x$	$F \rightarrow \bullet\, q$
$F \rightarrow q$	$F \rightarrow \bullet\, q$	$F \rightarrow q$	

The Algorithm, ctd

- Let x be an arbitrary terminal, A be an aribtrary nonterminal, and α and β be arbitrary (possibly empty) strings of symbols.
- In an item set i, take every production of the form $E \rightarrow \alpha \bullet x\beta$ and produce a new state j containing the transitive closure of $E \rightarrow \alpha x \bullet \beta$. Add a shift in the action table for column x and state i, and destination state j in the goto table for column x and state i.
- In an item set i, take every production of the form $E \rightarrow \alpha \bullet A\beta$ and produce a new state j containing the transitive closure of $E \rightarrow \alpha A \bullet \beta$. Add j to the goto table in column A and state i.
- In an item set i, take every rule of the form $E \rightarrow \alpha\bullet$ and add a reduce actions for state i for each terminal in the follow set of E.
- If an item set is recreated, reuse the original; do not create a duplicate.

Automata Example 1

Automata



Tabular Representation

Grammar

$S \rightarrow a\, b$

Action

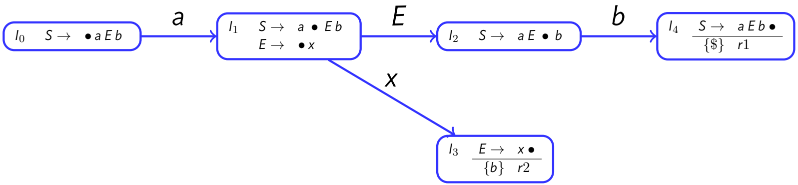
	a	b	$
0	s		
1		s	
2			r1

Goto

	a	b	$	S
0	1			
1		2		
2				

Automata Example 2

Automata



Grammar

$$S \to a\, E\, b$$
$$E \to x$$

Action

	a	b	x	$
0	s			
1			s	
2		s		
3		r2		
4				r1

Goto

	a	b	x	$	S	E
0	1					
1			3			2
2		4				
3						
4						

Automata Example 3.1

- Let's build the table for this automata.

$$\begin{array}{lcl} S & \rightarrow & a\,E\,b \\ & | & a\,b\,S \\ E & \rightarrow & E\,x \\ & | & b \end{array}$$

Automata Example 3.2

I_0	$S \rightarrow$	$\bullet\, a\, E\, b$
		$\bullet\, a\, b\, S$

Grammar

$$
\begin{aligned}
S &\rightarrow a\, E\, b \\
&\quad|\ a\, b\, S \\
E &\rightarrow E\, x \\
&\quad|\ b
\end{aligned}
$$

Action

	a	b	x	$
0				
1				
2				
3				
4				
5				
6				

Goto

	a	b	x	$	S	E
0						
1						
2						
3						
4						
5						
6						

Automata Example 3.2

$I_0 \quad S \to \bullet\, a\, E\, b \Leftarrow$
$\bullet\, a\, b\, S \Leftarrow$

Grammar

$$S \to a\, E\, b \mid a\, b\, S$$
$$E \to E\, x \mid b$$

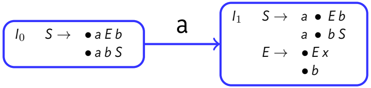
Action

	a	b	x	$
0				
1				
2				
3				
4				
5				
6				

Goto

	a	b	x	$	S	E
0						
1						
2						
3						
4						
5						
6						

Automata Example 3.3

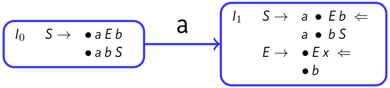


$$S \to a\,E\,b \mid a\,b\,S$$
$$E \to E\,x \mid b$$

	a	b	x	$
0	s			
1				
2				
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1						
2						
3						
4						
5						
6						

Automata Example 3.3

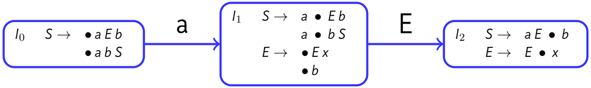


$$
\begin{aligned}
S &\rightarrow a\,E\,b \\
&\mid\; a\,b\,S \\
E &\rightarrow E\,x \\
&\mid\; b
\end{aligned}
$$

	a	b	x	$
0	s			
1				
2				
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1						
2						
3						
4						
5						
6						

Automata Example 3.4

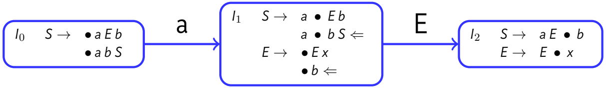


$$
\begin{aligned}
S &\to a\, E\, b \\
  &\mid a\, b\, S \\
E &\to E\, x \\
  &\mid b
\end{aligned}
$$

	a	b	x	$
0	s			
1				
2				
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1						2
2						
3						
4						
5						
6						

Automata Example 3.4

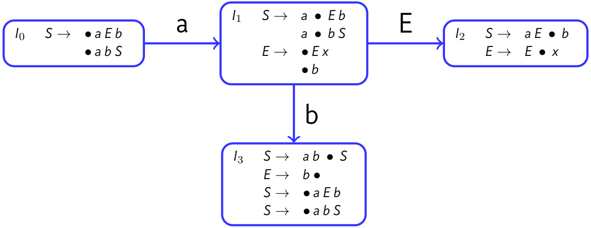


$$
\begin{aligned}
S &\to a\, E\, b \\
&\mid a\, b\, S \\
E &\to E\, x \\
&\mid b
\end{aligned}
$$

	a	b	x	$
0	s			
1				
2				
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1						2
2						
3						
4						
5						
6						

Automata Example 3.5

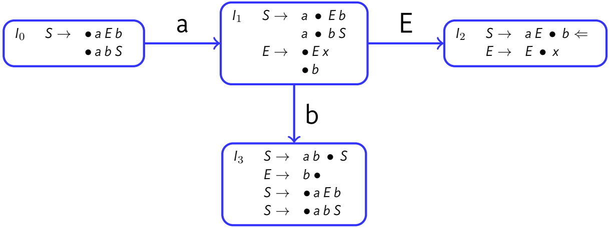


$$
\begin{aligned}
S &\to a\, E\, b \\
&\mid\ a\, b\, S \\
E &\to E\, x \\
&\mid\ b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2				
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2						
3						
4						
5						
6						

Automata Example 3.5

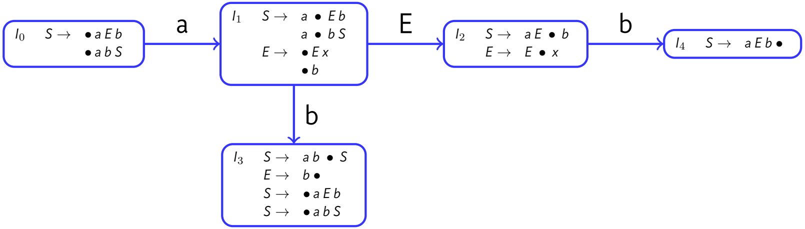


$$
\begin{aligned}
S &\to a\,E\,b \\
&\mid\ a\,b\,S \\
E &\to E\,x \\
&\mid\ b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2				
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2						
3						
4						
5						
6						

Automata Example 3.6

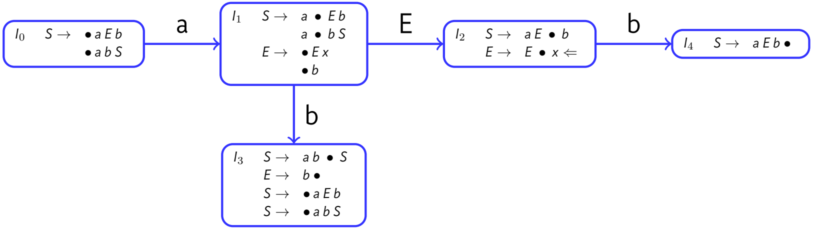


$$
\begin{aligned}
S &\to a\,E\,b \\
  &\mid a\,b\,S \\
E &\to E\,x \\
  &\mid b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3						
4						
5						
6						

Automata Example 3.6

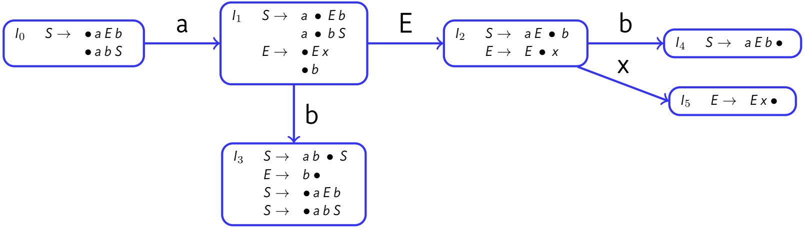


$$
\begin{aligned}
S &\to a\,E\,b \\
  &\mid a\,b\,S \\
E &\to E\,x \\
  &\mid b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3						
4						
5						
6						

Automata Example 3.7

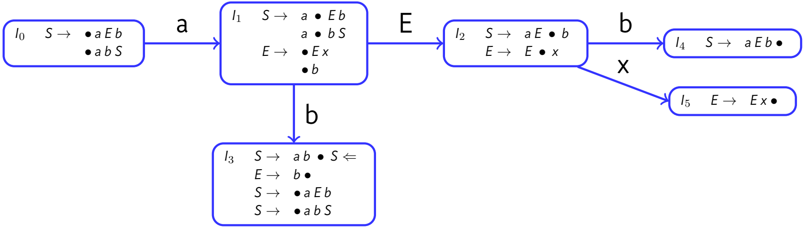


$$
\begin{aligned}
S &\to a\, E\, b \\
&\mid\ a\, b\, S \\
E &\to E\, x \\
&\mid\ b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3						
4						
5						
6						

Automata Example 3.7

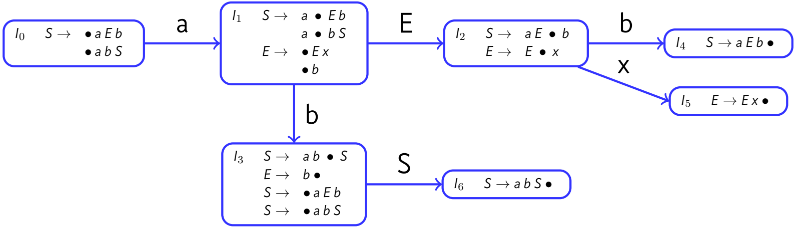


$$
\begin{aligned}
S &\rightarrow a\,E\,b \\
&\mid\ a\,b\,S \\
E &\rightarrow E\,x \\
&\mid\ b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3						
4						
5						
6						

Automata Example 3.8

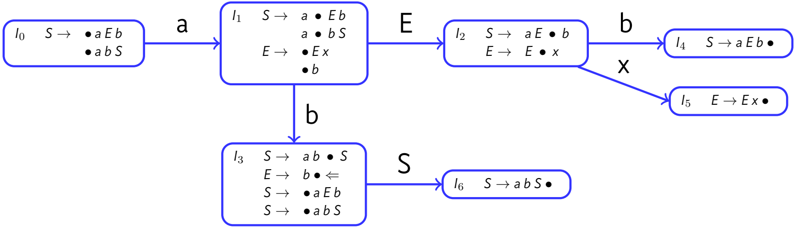


$$
\begin{aligned}
S &\to a\, E\, b \\
  &\mid\ a\, b\, S \\
E &\to E\, x \\
  &\mid\ b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3					6	
4						
5						
6						

Automata Example 3.8

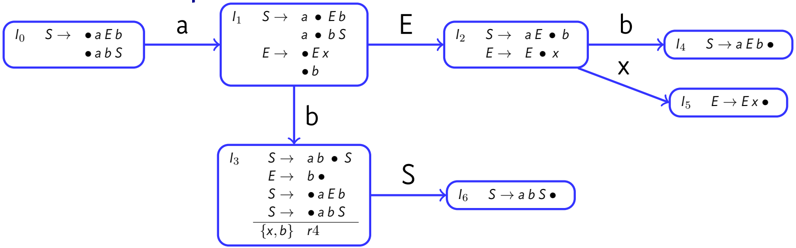


$$
\begin{array}{lcl}
S & \to & a\,E\,b \\
  & | & a\,b\,S \\
E & \to & E\,x \\
  & | & b
\end{array}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3				
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3					6	
4						
5						
6						

Automata Example 3.8

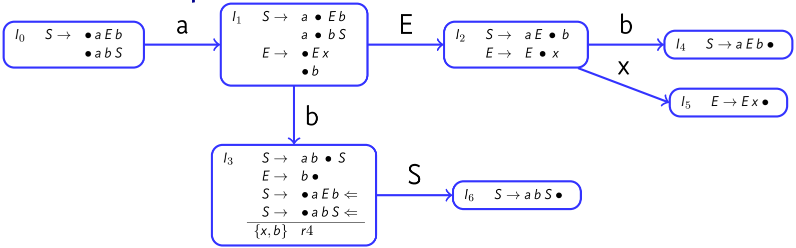


$$
\begin{aligned}
S &\to a\,E\,b \\
&\mid a\,b\,S \\
E &\to E\,x \\
&\mid b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3		r4	r4	
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3					6	
4						
5						
6						

Automata Example 3.8

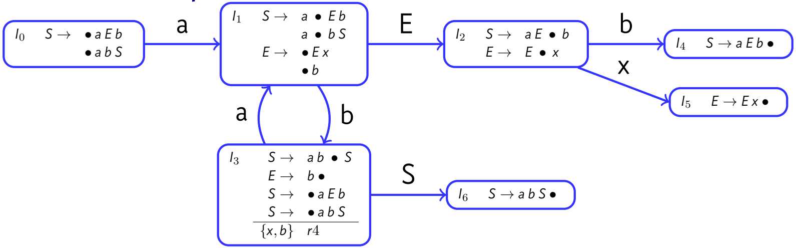


$$
\begin{aligned}
S &\to a\,E\,b \\
  &\mid\; a\,b\,S \\
E &\to E\,x \\
  &\mid\; b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3		r4	r4	
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3					6	
4						
5						
6						

Automata Example 3.9

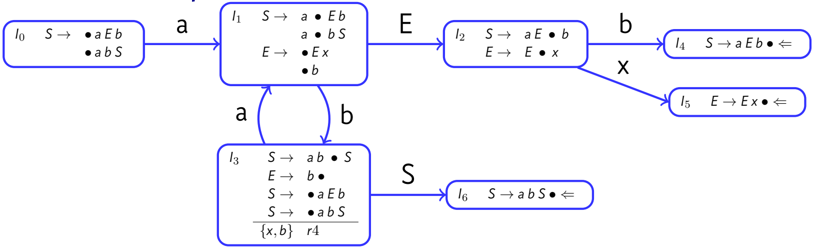


$$
\begin{aligned}
S &\rightarrow a\,E\,b \\
&\mid a\,b\,S \\
E &\rightarrow E\,x \\
&\mid b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3	s	r4	r4	
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3	1				6	
4						
5						
6						

Automata Example 3.9

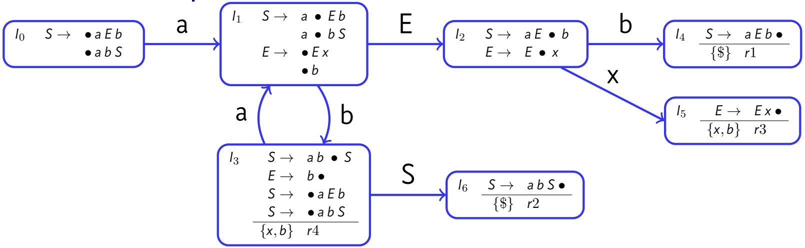


$$
\begin{aligned}
S &\to a\,E\,b \\
  &\mid\; a\,b\,S \\
E &\to E\,x \\
  &\mid\; b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3	s	r4	r4	
4				
5				
6				

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3	1				6	
4						
5						
6						

Automata Example 3.10



$$
\begin{aligned}
S &\to a\,E\,b \\
 &\mid a\,b\,S \\
E &\to E\,x \\
 &\mid b
\end{aligned}
$$

	a	b	x	$
0	s			
1		s		
2		s	s	
3	s	r4	r4	
4				r1
5		r3	r3	
6				r2

	a	b	x	$	S	E
0	1					
1		3				2
2		4	5			
3	1				6	
4						
5						
6						

Activity

Your turn. Try to build the automata for this grammar. There's a surprise waiting for you!

$$
\begin{array}{lcl}
S & \rightarrow & a\,E\,b \\
 & | & x \\
E & \rightarrow & E\,x\,E \\
 & | & b
\end{array}
$$