

Objectives

You should be able to ...

The Church-Rosser Property

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- ▶ Describe the Church-Rosser property.
- ▶ Explain the advantages it confers when a language has that property.

Other Arrow Notations

Notations

- $\rightarrow^0 \equiv$ The identity
- $\rightarrow^1 \equiv \rightarrow$
- $\rightarrow^n \equiv \rightarrow \cdot \rightarrow^{n-1}$
- $\rightarrow^* \equiv \bigcup_{i=0}^{\infty} \rightarrow^i$
- $\rightarrow^+ \equiv \bigcup_{i=1}^{\infty} \rightarrow^i$
- $a \leftarrow b \equiv b \rightarrow a$
- $\leftrightarrow \equiv \rightarrow \cup \leftarrow$
- $\leftrightarrow^* \equiv (\rightarrow \cup \leftarrow)^*$

Example

$3 \rightarrow^* 3$, and if $3 > 2$ then $5 + 9$ else $2 * 4 \rightarrow^* 14$

Be Careful with \leftrightarrow^*

$$a \leftrightarrow^* b \not\equiv a \leftarrow^* b \cup a \rightarrow^* b$$

For example $a \leftrightarrow^* b$ when

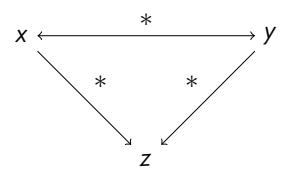
$$a \leftarrow a_1 \rightarrow a_2 \rightarrow a_3 \leftarrow b_2 \leftarrow b_1 \rightarrow b$$

Term Rewriting Systems

Transition semantics can be thought of as a *term rewriting system*. Common questions:

- ▶ Does an expression always terminate?
- ▶ Can we tell if two expressions are equal?

Church-Rosser property: if $x \leftrightarrow^* y$ then x and y normalize to the same value.



Who Has It?

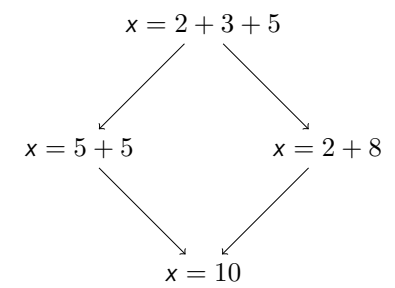
- ▶ Alonzo Church and J. Barkley Rosser proved that the λ -calculus has these properties in 1936.
- ▶ Very important for theorem provers
- ▶ Most programming languages have this property (some of the time).
- ▶ One Benefit: you can check for equality of x and y by evaluating them.



Example

Confluence

If $x \rightarrow y_1$ and $x \rightarrow y_2$ then y_1 and y_2 normalize to the same value. (Confluence and the Church-Rosser property coincide.)



This is also known as the "diamond property."

