The Church-Rosser Property

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Objectives

You should be able to ...

- ► Describe the Church-Rosser property.
- Explain the advantages it confers when a language has that property.

Other Arrow Notations

Notations

$$\begin{array}{cccc} \rightarrow^{0} & \equiv & \text{The identity} \\ \rightarrow^{1} & \equiv & \rightarrow \\ \rightarrow^{n} & \equiv & \rightarrow \cdot \rightarrow^{n-1} \\ \rightarrow^{*} & \equiv & \bigcup_{i=0}^{\infty} \rightarrow^{i} \\ \rightarrow^{+} & \equiv & \bigcup_{i=1}^{\infty} \rightarrow^{i} \\ a \leftarrow b & \equiv & b \rightarrow a \\ \leftrightarrow & \equiv & \rightarrow \cup \leftarrow \\ \leftrightarrow^{*} & \equiv & (\rightarrow \cup \leftarrow)^{*} \end{array}$$

Example

 $3 \rightarrow^* 3$, and if 3 > 2 then 5 + 9 else $2 * 4 \rightarrow^* 14$

Be Careful with \leftrightarrow^*

$$a \leftrightarrow^* b \not\equiv a \leftarrow^* b \cup a \rightarrow^* b$$

For example $a \leftrightarrow^* b$ when

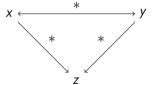
$$a \leftarrow a_1 \rightarrow a_2 \rightarrow a_3 \leftarrow b_2 \leftarrow b_1 \rightarrow b$$

Term Rewriting Systems

Transition semantics can be thought of as a *term rewriting system*. Common questions:

- ► Does an expression always terminate?
- Can we tell if two expressions are equal?

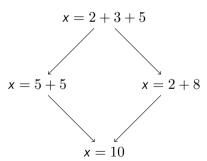
Church-Rosser property: if $x \leftrightarrow^* y$ then x and y normalize to the same value.



Example

Confluence

If $x \to y_1$ and $x \to y_2$ then y_1 and y_2 normalize to the same value. (Confluence and the Church-Rosser property coincide.)



This is also known as the "diamond property."

Who Has It?

- Alonzo Church and J. Barkley Rosser proved that the λ -calculus has these properties in 1936.
- Very important for theorem provers
- Most programming languages have this property (some of the time).
- ▶ One Benefit: you can check for equality of *x* and *y* by evaluating them.